Pharmaceutical statistics

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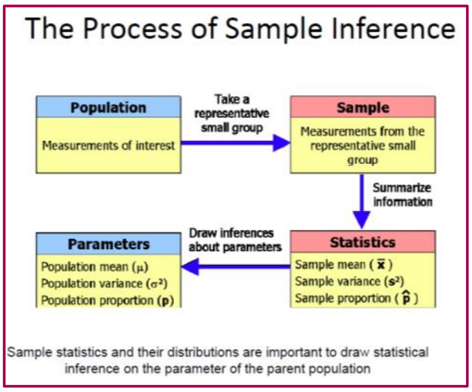
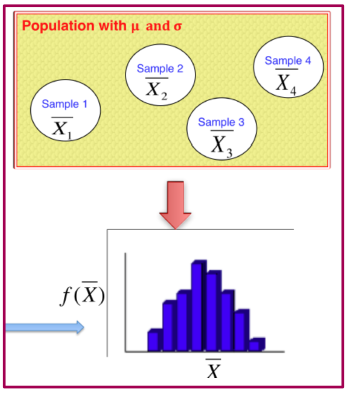
ARKAN
ACADEMY

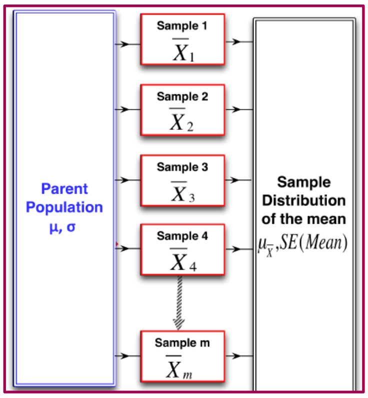
Sampling Distribution of the mean

- ***Sampling Distribution of a Sample Statistic***: The distribution of values for a sample statistic obtained from repeated samples, all of the same size and all drawn from the same population.
  - *Population* is all the elements that are included in our study.
  - The sample is a subset that is chosen randomly from that population
  - To make inferences about a population from a repeated number of samples.
  - I will take all possible samples, calculate their mean ($\bar{x}$), then make distribution graph (($\bar{x}$) and f($\bar{x}$) we call it *sampling distribution of the mean.*
  - Samples mean can vary from sample to sample. Since X is a number and it can change from each time, it can be thought of as a random variable.

- How are statistics (means) distributed over many samples?
  - The sampling distribution of a statistic is the probability distribution for the values of the statistic that results when random samples of size n are repeatedly drawn from the population.
  - Sampling distribution is just a *probability distribution* that involves a statistic.
  - *The standard error* is the standard deviation of the sampling distribution of a statistic and is represented by the symbol SE (mean).



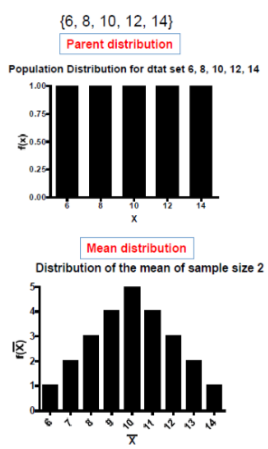




- ***Sampling With Replacement***
  - For a population set of data: 6, 8, 10, 12, 14
  - μ = 10, σ^2 = 8
  - We want to construct a sampling distribution of the mean using sampling size of 2.
  - The number of different samples (all possible sample size) is (N^n) = 5^2 = 25
  - $\mu_{\bar{X}} = \mu = 10$
  - $\sigma^2_{\bar{x}} = \dfrac{\sigma^2}{n}$ = 8/2 = ***4***
  - $\sigma_{\bar{X}} = SE = \sqrt{\dfrac{\sigma^2}{n}} = \dfrac{\sigma}{\sqrt{n}}$ = ***2*** Note that: ($\sigma_{\bar{x}} < \sigma$)

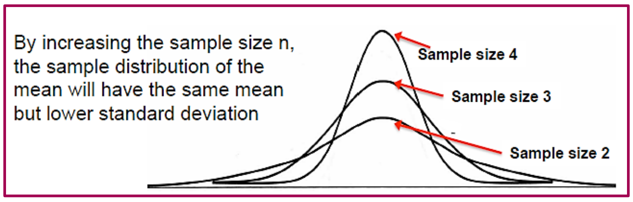
6,6 (6)	6,8 (7)	6,10 (8)	6,12 (9)	6,14 (10)
8,6 (7)	8,8 (8)	8,10 (9)	8,12 (10)	8,14 (11)
10,6 (8)	10,8 (9)	10,10 (10)	10,12 (11)	10,14 (12)
12,6 (9)	12,8 (10)	12,10 (11)	12,12 (12)	12,14 (13)
14,6 (10)	14,8 (11)	14,10 (12)	14,12 (13)	14,14 (14)

Sample mean (X)	Frequency	Relative Frequency
6	1	1/25
7	2	2/25
8	3	3/25
9	4	4/25
10	5	5/25
11	4	4/25
12	3	3/25
13	2	2/25
14	1	1/25
Total	25	25/25

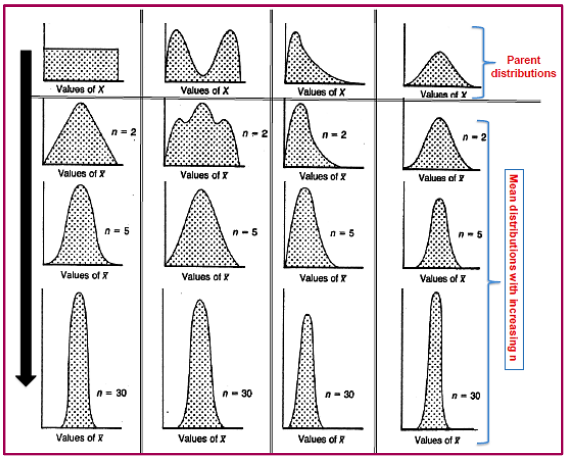


  - By increasing the sample size n, the sample distribution of the mean will have the same mean but ***lower*** standard deviation.

Sample size	# possible samples (N^n)	Mean of samples ($\mu_{\bar{X}}$)	Standard deviation of samples $\sigma_{\bar{X}}$
2	25	10	2
3	125	10	1.63
4	625	10	1.41



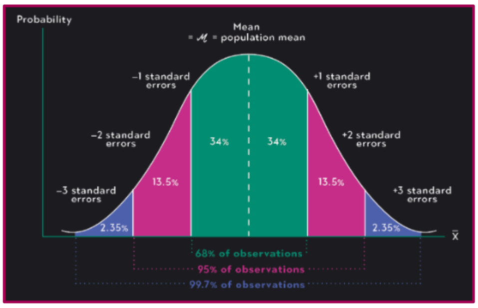
- ***Sampling from Nonnormally Distributed Populations***



  - For the case where sampling is from a *nonnormally distributed population*, we refer to an important mathematical theorem called the *central limit theorem*.
  - Given a population of any nonnormal functional form with a mean μ and a variance σ^2, the sampling distribution computed from a sample size n from this population, will have mean μ and a variance σ^2/n and will be approximately normally distributed <u>when the sample size is large</u>. As sample size is increased the distribution becomes <u>more normal</u> and its standard deviation is decreased.
  - If the population is not normal distributed, then to get normal sample distribution you should take a sample Size ≥ 30.

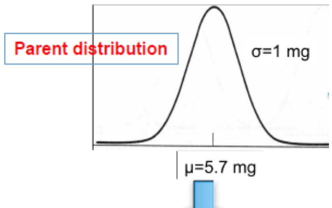
- ***Summary***
  - The mean of the sampling distribution of has the same value as the mean of the original population μ
  - The variance of the sampling distribution of the mean is *not equal* to the population variance, instead it is equal to the population variance divided by the size of the sample used to obtain the sampling distribution.
  - The square root of the variance of the sampling distribution is called the *standard error of the mean* or, simply, the standard error. SE(Mean).

- ***Why do we want the sampling distribution of the mean to be normal??***
  - ***Easier to Make Inferences:***

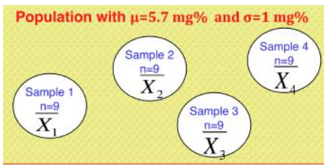


    - ✓ The normal distribution has well-known properties, such as symmetry and predictable proportions (e.g., 68%, 95%, 99.7% for 1, 2, and 3 standard deviations, respectively).
    - ✓ This allows us to calculate probabilities.
    - ✓ We can apply all the rules of normal distribution on the sampling distribution of the mean.

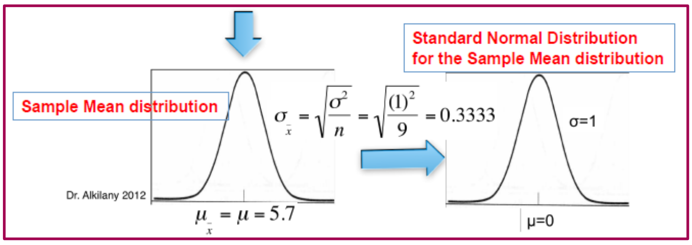
  - ★ Example 1:



    - ✓ If uric acid values in normal adult males are approximately *normally distributed* with a mean and standard deviation of 5.7 & 1 mg%, respectively.
    - ✓ Find the probability that a *sample of size* **9** will yield a *mean*:
      1) Greater than 6 mg %
      2) Between 5 and 6 mg %
      3) Less than 5.2 mg%



    - ✓ The mean and the standard deviation of the sampling distribution (n=9) is: $\mu_{\bar{x}} = \mu = 9$
    - ✓ Now we deal with the sample mean distribution and we will find the z-score for mean=6 and use the table as we did before…..



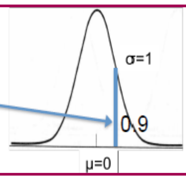
    - ✓ $Z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$

- **Q1:** Find the probability that a *sample of size* **9** will yield a *mean* greater than 6 mg %

$$z_{\bar{x}} = \frac{6 - 5.7}{0.3333} = 0.90$$

P(X≥6) =P(Z≥0.9)=1-P (z ≤ 0.9)

= 1 −0.8159=0.1841



- **Q2:** Find the probability that a *sample of size* **9** will yield a *mean* between 5 and 6 mg %

$$Z_6 = \frac{6 - 5.7}{0.3333} = 0.90$$

$$Z_5 = \frac{5 - 5.7}{0.3333} = -2.1$$

$P(-2.1 \le z \le 0.9) = P(Z \le 0.9) - P(Z \le -2.1) = 0.8159 - 0.0179 = 0.798$

- Find the probability that a *sample of size* **9** will yield a *mean* less than 5.2 mg%

$$Z_{5.2} = \frac{5.2 - 5.7}{0.3333} = -1.5$$

$P(Z<1.5) = 0.0668$

★ Example 2:

- If the mean and standard deviation of serum iron values for healthy men are 120 µg/100 ml and 15 µg/100 mL, respectively, what is the probability that a random sample of 50 normal men will yield a mean between 115 and 125 µg/100 mL?
- μ=120 µg/100 mL
- σ =15 µg/100 mL
- Sample size: n=50
- SE = 2.121µg/100mL.
- Find $P(115 < \bar{x} < 125)$:

  $Z_{115} = -2.36$

  $Z_{125} = 2.36$

  $P(Z \le 2.36) - P(Z \le 2.36) = 0.9909 - 0.0091 = 0.9818.$

★ Example 3:

- Consider a normal population with mean $\mu = 50$ and $\sigma = 15$. Suppose a sample of size 9 is selected at random. SE = 5
- Find:
- $P(45 \le x \le 60)$

  $P(Z \le 2) - P(Z \le 1) = 0.97 - 0.16 = 0.81.$
- $P(x \le 47.5)$

  $Z = -0.5$

  $P(Z \le -0.5) = 0.3085$

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